

Chapter 1. Relations and Functions

General direction for the students :-Whatever be the notes provided , everything must be copied in the Maths Copy and then do the Home work in the same Copy.

Relation (R) on a set A:-

Means subset of A cross A . Where 'A' is any non-empty set.

*If a set 'A' is having 'n' elements , then the total number of relations on 'A' is $2^{n \times n}$.

Types of Relation(R) on a set A.

i) Reflexive Relation:- It is a relation of an element with itself.

For example : A triangle ' is congruent ' with itself.

Algebraically we can write xRx ie $(x, x) \in R \quad \forall x \in A$

ii) Symmetric Relation:- It is the relation between two elements in such a way that,

if x related y then y is also related with x with same relation.

For example : If a line l is perpendicular to the line m, then m is also perpendicular to l.

Algebraically we can write

If xRy then yRx ie If $(x, y) \in R$ then (y, x) is also belongs to $R \quad \forall x, y \in A$.

iii) Transitive Relation:- It is the relation between three elements in such a way that

if xRy and yRz then $xRz \quad \forall x, y, z \in A$.

ie If $(x, y) \in R$ and $(y, z) \in R$ then (x, z) is also belongs to $R \quad \forall x, y, z \in A$.

For example : For triangle T_1 , T_2 and T_3 . If a triangle T_1 is similar to T_2 and T_2 is similar to T_3 , then T_1 is also similar to T_3 .

Equivalence Relation:- A relation (R) on a non-empty set is called Equivalence relation iff it is Reflexive , Symmetric and Transitive.

Eg; '*is similar*' in set of all triangles in a plane is an equivalence relation.

Domain:- set of all first components (x) of the ordered pairs.

Range:- Set of all second components (y) of the ordered pairs.

Codomain:- If R is from A to B , then B is called Codomain of R.

NOTE: '*x congruent y modulo m written as $x \equiv y \pmod{m}$* ' means $x - y$ is divisible by m. where $x, y \in Z$ and $m \in Z^+$.

Exercise 1.1

Q1 i) Given $R: Z \rightarrow Z$

$$R = \{(x, y): x - y \text{ is an integer}\}$$

We know, $x - x = 0$ is an integer $\forall x \in Z$

$\Rightarrow R$ is reflexive relation.

Let $x, y \in Z$

$\Rightarrow x - y$ is an integer

Also $y - x$ is an integer

$\Rightarrow R$ is symmetric relation.

Let $x, y, z \in Z$

$\Rightarrow x - y$ and $y - z$ are integers

Also $x - z$ is an integer

$\Rightarrow R$ is Transitive relation

Q8) We know from Geometry that , any triangle is congruent with itself.

\Rightarrow the relation is Reflexive.

Let T_1, T_2 are any two triangles in a plane

We know from Geometry that , if $T_1 \cong T_2$, then $T_2 \cong T_1$

\Rightarrow the relation is Symmetric.

Let T_1, T_2 and T_3 are any three triangles in a plane

We know from Geometry that , if $T_1 \cong T_2$ and $T_2 \cong T_3$ then $T_1 \cong T_3$

\Rightarrow the relation is Transitive .

Since the relation is Reflexive, Symmetric and Transitive , it is an Equivalence relation.

Q16) Since A is having 3 elements , number of relations on A is $2^{3 \times 3} = 512$.

Q22) Let us consider three real numbers $\frac{1}{2}, -\frac{1}{3}$ and -4

$$A/Q \quad 1 + \frac{1}{2} \times -\frac{1}{3} = \frac{5}{6} > 0 \quad \text{again } 1 + -\frac{1}{3} \cdot -4 = \frac{7}{3} > 0$$

$$\text{But } 1 + \frac{1}{2} \cdot -4 = -1 < 0$$

⇒ the relation is not transitive.

HOME WORK : Exercise 1.1, Question numbers 2 , 6 , 7 9 , 10 , 14 ,15, 17 , 19 ,20, 21, 24 and 25.
